Real Option Valuation as a Modelling Problem

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Lappeenranta University of Technology (LUT) "system"

LUT is a technical university that focuses on the nexus of technology and business

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3 Faculties
This presentation

• Introduction to Real Options
• Real Option valuation as a modeling problem
• Four selected models for real option valuation and more on one of them
• Some thoughts for future research on real options & real option models
Real Options

• Real options are investment possibilities and possibilities within investment projects
• Real option valuation is valuing these possibilities as ”real world” options
• Valuation started by using the same methods that have been used for financial option valuation
• Now some models designed especially for real option valuation methods have emerged
Example: Commercial space

A company needs new premises as the operations have grown fast, they decide to build a new commercial space:

The markets for the company product are cyclical and the need for space is variable depending on the season.

They decide to build real options into the spaces to help!

We build a part of the space into a separate module with a separate entrance and so that it can be easily separated from the rest of the spaces: with for example a removable wall.

We can go even further and build more than one separable module into the space. This will lead to even more flexibility. But will cost more to build.

The value of the possibility to rent the (extra) spaces is the value of the real option. This changes according to the markets.
Real Options Analysis

- Used commonly to frame strategic investments:
  - R&D projects / portfolios
  - Patents / immaterial rights
  - Real Estate (land value)
- Used also as a metric in comparison of competing projects
- ”Projects with negative expected value may still be worth something”

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During the option maturity the value of the option may vary. A process is used to "create" the distribution of outcomes.

The single value representing the future value distribution is calculated by using the likelihood of occurrence of (each of) the values as a weight for the value, such that all negative values are assigned value zero.

These values are assigned the value zero.

Discounting the expected value to present value
Real Options as a Modeling problem

The three major components of modeling the value of a (real) option are:

a) the modeling of the future value distribution

b) the calculation of the expected value of the future value distribution while mapping negative values of the distribution zero, and

c) modeling the calculation of the present value of the expected value.
Real Options as a Modeling problem

The reality in which real options exist:

- No established or incomplete & inefficient markets
- Arbitrage possibilities
- Stakeholders have possibilities to affect real option value
- Informational assymmetry
- Often no historical data available
"Multiple sigma events not at all impossible"
Four model types for real option valuation

There are four types of models for the valuation of real options currently in ”main stream use”

• Differential equation solutions
• Discrete event and decision models
• Simulation based models
• Fuzzy logic based methods

The modeling choices of these models are of interest here: How do the different model types ”see” the same issue?
Four model types for real option valuation

For the purpose of looking at this question I have selected one model from each group:

• The Black & Scholes model
• The binomial option valuation model
• Datar-Mathews model for ROV
• (fuzzy) Pay-off method for ROV
Black & Scholes Model (1973)

• A stochastic (GBM) process used to yield a continuous log-normal distribution of future asset value
• Calculation of expected value as a probability weighted average of the positive side of the future value distribution
• Discounting the expected value to present value with the risk-free rate of return, continuous compounding
• Same discount rate used for revenue & cost
• Input parameters fed into a formula – out pops the answer
• Closed form solution, based on the replication argument: Any two assets that yield same cash-flows under the same circumstances must be worth the same price
• Mathematically very elegant
• Based on a strict set of assumptions about markets (complete / efficient)
Black & Scholes Model (1973)

\[ C = SN(d_1) - Xe^{-r(T-t)}N(d_2) \]

\[ d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \]

\[ d_2 = d_1 - \sigma\sqrt{T-t} \]

Where:
- \( C \) is the European Call option price
- \( S \) is the price of the underlying asset
- \( X \) is the exercise price
- \( T-t \) is the time to maturity
- \( r \) is the risk-free rate of return
- \( \sigma \) is the volatility
- \( N \) is the cumulative normal distribution function

Variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result presented as a single number</td>
<td></td>
</tr>
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</table>

Variable values

Formula

Result

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Binomial Model (1979)

- A binomial tree process used to create a future value distribution
- Backwards iteration to find the option value
- Iteration includes discounting with a compounding risk-free rate of return for cost and for revenues
- With a large number of time steps the result converges with the result from the Black & Scholes model
- Strict assumptions about the markets
- Input info for standard deviation and up/down probability needed

Result presented as a single number

• Future value distribution creation based on using manager-created cash-flow scenarios for a project as input to a Monte Carlo simulation that creates a probability distribution
• Expected value calculated as the probability weighted average of the positive side of the future value distribution
• Discount rates may be separate for cost and revenue CF, selectable compounding interval
• No strict assumptions about the markets / reality
• Designed for spread-sheet software
• Same answer as Black & Scholes reachable with same assumptions

Cash-flow scenarios ➔ Monte Carlo Simulation ➔ distribution ➔ Result

Result presented as a single number
Pay-off method for real option valuation (2009)

- Manager created cash-flow scenarios are used to create (simplified) future value distributions that are treated as fuzzy numbers (often triangular / trapezoidal)
- The "expected value" is calculated from the fuzzy numbers as the possibilistic mean of the positive side of the fuzzy number
- Discounting is done by using separate discount rates for revenues and costs
- Compounding interval at the discretion of the user
- No strict assumptions about the markets / reality
- Spread-sheet compatible
- Designed for the practitioner – "as easy as possible"
Pay-Off Method Shortly

The method of calculation can also be "less" standard. Many times cash-flows cannot be estimated, only "larger" aggregates are available.

<table>
<thead>
<tr>
<th>Strategic patent portfolio 1</th>
<th>Min possible</th>
<th>Best estimate</th>
<th>Max possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total size of future markets ($)</td>
<td>$900M</td>
<td>$1700M</td>
<td>$2900M</td>
</tr>
<tr>
<td>Market share (%)</td>
<td>6%</td>
<td>15%</td>
<td>24%</td>
</tr>
<tr>
<td>Likelihood of enabling portfolio (%)</td>
<td>30%</td>
<td>40%</td>
<td>55%</td>
</tr>
<tr>
<td>Overall chances (%)</td>
<td>75%</td>
<td>90%</td>
<td>100%</td>
</tr>
<tr>
<td>V_{Portfolio} = TSFM<em>EMS</em>LBEP*RT</td>
<td>$12,15M</td>
<td>$306M</td>
<td>$382,8M</td>
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<tr>
<td>PV total costs of portfolio</td>
<td>$8M</td>
<td>$14M</td>
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<tr>
<td>NPV_portfolio</td>
<td>$-22,85M</td>
<td>$292M</td>
<td>$374,8M</td>
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</table>

CUMULATIVE NET PRESENT CASH-FLOW SCENARIOS FOR THE ASSET

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>maximum</td>
<td>-800,00</td>
<td>-452,17</td>
<td>-149,72</td>
<td>113,29</td>
<td>341,99</td>
<td>540,81</td>
<td>713,75</td>
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<tr>
<td>Best est.</td>
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<td>-632,34</td>
<td>-467,96</td>
<td>-325,03</td>
<td>-200,76</td>
<td>-92,05</td>
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<tr>
<td>minimum</td>
<td>-1000,00</td>
<td>-913,04</td>
<td>-761,81</td>
<td>-630,31</td>
<td>-515,96</td>
<td>-416,51</td>
<td>-330,08</td>
</tr>
</tbody>
</table>

"EACH SCENARIO IS JUST TRADITIONAL NPV "
"Everyone in business knows this!"

We ask for three scenarios:
1. Best estimate
2. minimum possible
3. maximum possible
These can be fully unrelated or fully related to each other.
Pay-Off Method Shortly

1. Build 3-4 cash-flow scenarios and perform present value & NPV calculations

2. Create the pay-off distribution

3. Calculate descriptives for additional decision-support

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Pay-Off Method Shortly

1. Build 3-4 cash-flow scenarios and perform present value & NPV calculations

2. Create the pay-off distribution

3. Calculate descriptives for additional decision-support

It is not suggested that this is a perfect reflection of reality, or that this would be a very accurate description of reality. This is a way to graphically represent inaccurate knowledge that certainly looks very interesting!

As a result we have created a distribution of the NPV of the asset, based on the Information that we have.

The three scenarios' for the NPV of the asset mapped on a $ (vertical) axis.

We do not consider outcomes outside the min and max possible!

On the horizontal axis we map the degree of membership in the (set of) possible outcomes.

1 = fully possible outcome
0 = not a possible outcome

The most likely scenario is assigned full membership in the set of possible outcomes.
Pay-Off Method Shortly

1. Build 3-4 cash-flow scenarios and perform present value & NPV calculations

2. Create the pay-off distribution

3. Calculate descriptives for additional decision-support

"If we turn the problem" and look at it from a different angle...

We can treat the resulting distribution as a fuzzy number. This allows us to calculate interesting descriptive numbers directly from it.

With simple calculations we can calculate, for example: a real option value and a (possibilistic) mean for the asset.

These will help us better understand the value of the assets to the firm, which is our goal.
Real Option Valuation based on a pay-off distribution

\[ ROV = \frac{\int_0^\infty A(x)\,dx}{\int_{-\infty}^\infty A(x)\,dx} \times E(A_+) \]

Possibilistic mean of the positive side of the distribution.

Simple formulae, used based on where zero “crosses” the distribution
Comparison of assets becomes more intuitively understandable

<table>
<thead>
<tr>
<th>ASSET 1</th>
<th>ASSET 2</th>
<th>ASSET 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="graph1.png" alt="Graph" /></td>
<td><img src="graph2.png" alt="Graph" /></td>
<td><img src="graph3.png" alt="Graph" /></td>
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</table>

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<tr>
<th></th>
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<th>Best guess</th>
<th>Best guess</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>ASSET 1</strong></td>
<td>238850</td>
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<td>215309</td>
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<tr>
<td><strong>ASSET 2</strong></td>
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<td>387400</td>
<td>-127600</td>
<td>191383</td>
<td>206633</td>
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<tr>
<td><strong>ASSET 3</strong></td>
<td>89650</td>
<td>333650</td>
<td>23544</td>
<td>119299</td>
<td>119299</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The decision-maker will have to make a "real" decision = selection!

Focus on potential! Focus on downside! Quantifying uncertainty!
## Models summary

<table>
<thead>
<tr>
<th></th>
<th>Process to create future value distribution</th>
<th>Calculation of expected value</th>
<th>Discounting</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black &amp; Scholes</td>
<td>GBM, continuous log normal</td>
<td>Probability weighted average</td>
<td>Continuous with rf (same for R &amp; C)</td>
<td>Closed form solution, replication</td>
</tr>
<tr>
<td>Binomial tree model</td>
<td>Binomial tree, quasi log normal</td>
<td>Probability weighted average</td>
<td>Compound interval the time step used, risk free rate</td>
<td>Backwards iteration to solve value, approaches B&amp;S</td>
</tr>
<tr>
<td>Datar – Mathews</td>
<td>Managerial CF scenarios =&gt; Monte Carlo</td>
<td>Probability weighted average</td>
<td>Separate rd for cost &amp; revenue, c. interval selectable</td>
<td>Practitioner oriented</td>
</tr>
<tr>
<td>Fuzzy pay-off method</td>
<td>Managerial CF scenarios =&gt; fuzzy number</td>
<td>Possibilistic mean value</td>
<td>Separate rd for cost &amp; revenue, c. Interval selectable</td>
<td>Simple to use, fuzzy logic</td>
</tr>
</tbody>
</table>
Thoughts about future research
Thoughts about future research

Using **subjective probability** in framing the problem
- similar ease of use obtainable as with the fuzzy pay-off method, but with probability theory
- same methods for creation of cash-flow scenarios etc. apply – just a different choice of theory for treatment of uncertainty
Thoughts about future research

Using **function recognition** / estimation for deriving a ”high fit” function (curve) for a distribution based on managerial information - managers draw a future value distribution and curve is fitted to allow ”normal” probability distribution based real option valuation

A bit ”out there” approach
Thoughts about future research

Presentation of results is a KEY factor in successful use of real options!
And mind you ANY OR Results!!!
Research into *how results are presented* ”does not exist” in connection with real option valuation
This is something that I see as a modellers’ failure – only the valuation is modelled, but presenting the results is left for others to ponder.
Final thoughts

• I am quite sure that there are numerous new possibilities to model real option valuation – the problem is good and relevant and there is a real need from the practitioners

• Presentation of results is also something that merits the modelers’ time => This is AS IMPORTANT as the model itself, perhaps even more important!

• How much time did you ever think about presentation of your results?
New book out on the pay-off method!
Thank you!

Comments?
Observations?
Questions?
References

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