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MODELING REAL OPTIONS

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# Real Option Valuation as a Modelling Problem

Mikael Collan
Professor of Accounting
Lappeenranta University of Technology
Lappeenranta, Finland
mikael.collan@lut.fi

# Lappeenranta University of Technology (LUT) ”system”


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# This presentation

- Introduction to Real Options
- Real Option valuation as a modeling problem
- Four selected models for real option valuation and more on one of them
- Some thoughts for future research on real options & real option models

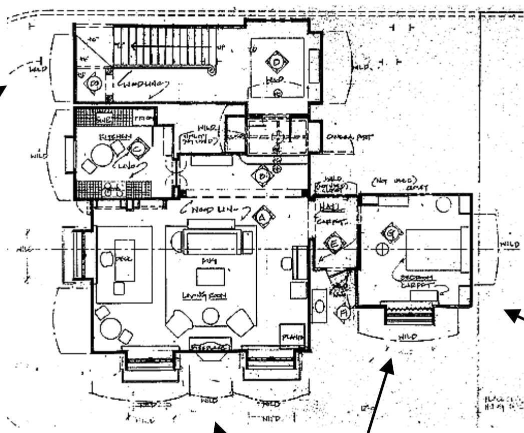
# Real Options

- Real options are investment possibilities and possibilities within investment projects
- Real option valuation is valuing these possibilities as ”real world” options
- Valuation started by using the same methods that have been used for financial option valuation
- Now some models designed especially for real option valuation methods have emerged

simple

# Example: Commercial space

**A company needs new premises as the operations have grown fast, they decide to build a new commercial space:**



The markets for the company product are cyclical and the need for space is variable depending on the season

They decide to build real options into the spaces to help!

We build a part of the space into a separate module with a separate entrance and so that it can be easily separated from the rest of the spaces: with for example a removable wall.

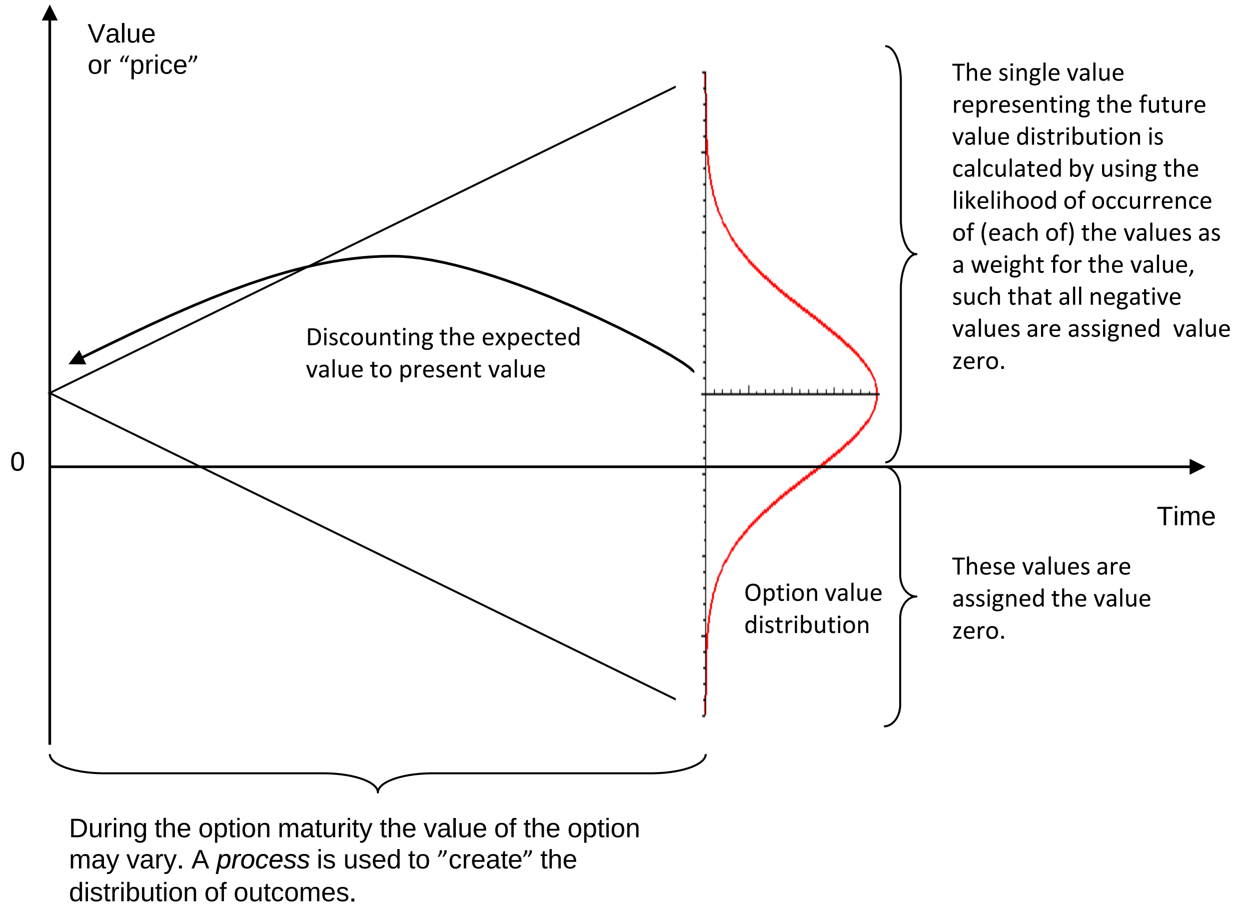
We can go even further and build more than one separable module into the space. This will lead to even more flexibility. But will cost more to build.

The value of the possibility to rent the (extra) spaces is the value of the real option. This changes according to the markets.

# Real Options Analysis

- Used commonly to frame strategic investments:
  - R&D projects / portfolios
  - Patents / immaterial rights
  - Real Estate (land value)
- Used also as a metric in **comparison** of competing projects
- ”Projects with negative expected value may still be worth something”

# Option valuation logic



# Real Options as a Modeling problem

The three major components of modeling the value of a (real) option are:

a) the modeling of the future value distribution

b) the calculation of the expected value of the future value distribution while mapping negative values of the distribution zero, and

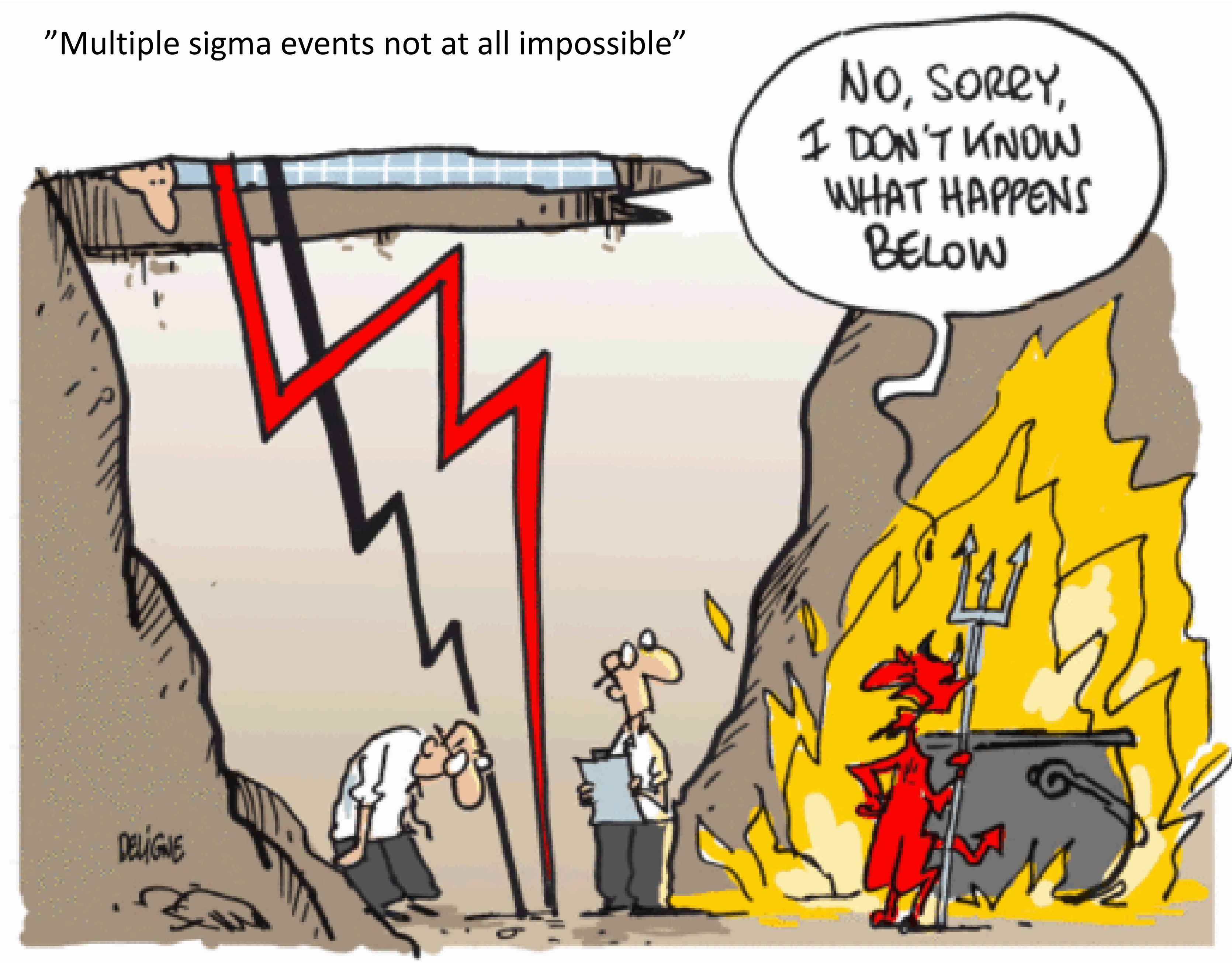
c) modeling the calculation of the present value of the expected value.

# Real Options as a Modeling problem

The reality in which real options exist:

- No established or incomplete & inefficient markets
- Arbitrage possibilities
- Stakeholders have possibilities to affect real option value
- Informational assymmetry
- Often no historical data available

"Multiple sigma events not at all impossible"



# Four model types for real option valuation

There are four types of models for the valuation of real options currently in ”main stream use”

- Differential equation solutions
- Discrete event and decision models
- Simulation based models
- Fuzzy logic based methods

The modeling choices of these models are of interest here: How do the different model types ”see” the same issue?

# Four model types for real option valuation

For the purpose of looking at this question I have selected one model from each group:

- The Black & Scholes model
- The binomial option valuation model
- Datar-Mathews model for ROV
- (fuzzy) Pay-off method for ROV

# Black & Scholes Model (1973)

- A stochastic (GBM) process used to yield a continuous log-normal distribution of future asset value
- Calculation of expected value as a probability weighted average of the positive side of the future value distribution
- Discounting the expected value to present value with the risk-free rate of return, continuous compounding
- Same discount rate used for revenue & cost
- Input parameters fed into a formula – out pops the answer
- **Closed form solution**, based on the replication argument: Any two assets that yield same cash-flows under the same circumstances must be worth the same price
- Mathematically very elegant
- Based on a strict set of assumptions about markets (complete / efficient)

# Black & Scholes Model (1973)

$$C = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + ½\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

| | |
|---|---|
| C | is the European Call option price |
| S | is the price of the underlying asset |
| X | is the exercise price |
| T-t | is the time to maturity |
| r | is the risk-free rate of return |
| σ | is the volatility |
| N | is the cumulative normal distribution function |

Variable values

Formula

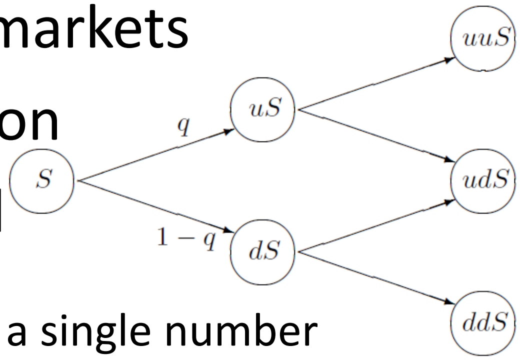
Result

Result presented as a single number

# Binomial Model (1979)

- A binomial tree process used to create a future value distribution
- Backwards iteration to find the option value
- Iteration includes discounting with a compounding risk-free rate of return for cost and for revenues
- With a large number of time steps the result converges with the result from the Black & Scholes model
- Strict assumptions about the markets
- Input info for standard deviation

and up/down probability needed

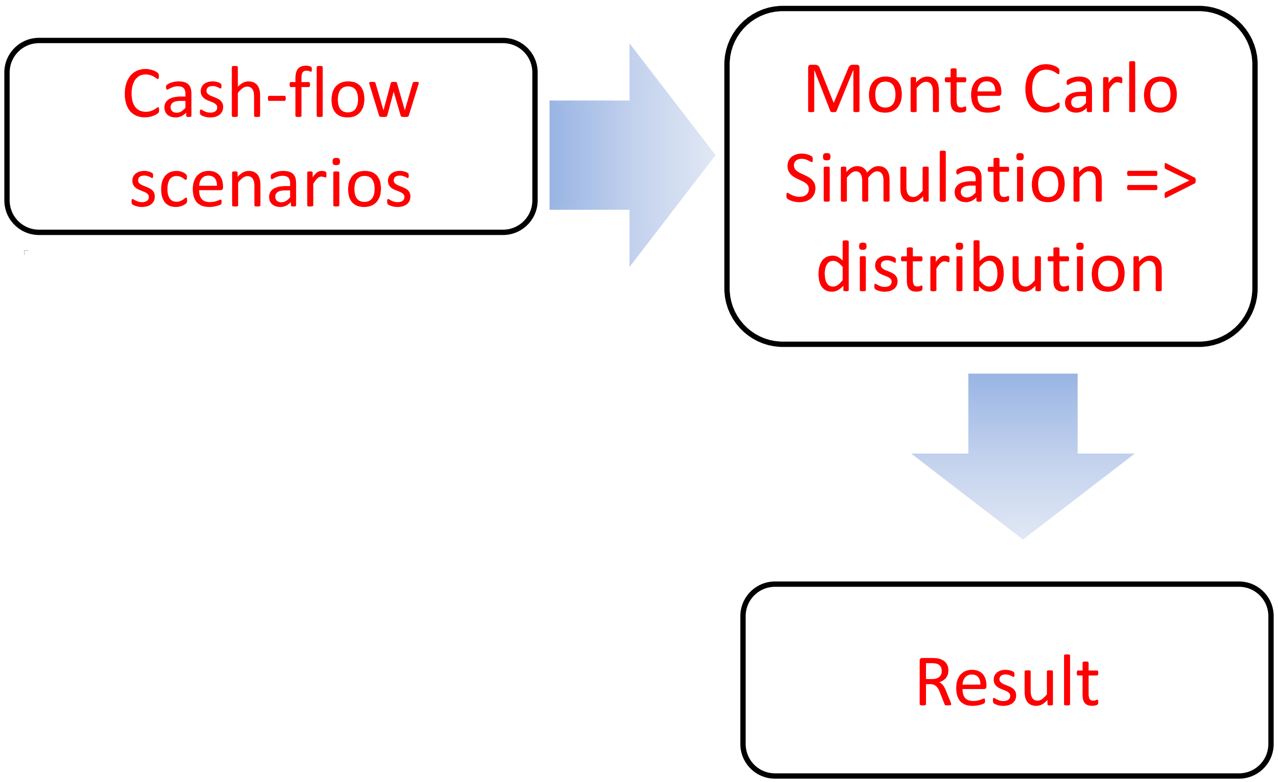



Result presented as a single number

# Datar – Mathews model (2004)

- Future value distribution creation based on using manager-created cash-flow scenarios for a project as input to a Monte Carlo simulation that creates a probability distribution
- Expected value calculated as the probability weighted average of the positive side of the future value distribution
- Discount rates may be separate for cost and revenue CF, selectable compounding interval
- No strict assumptions about the markets / reality
- Designed for spread-sheet software
- Same answer as Black & Scholes reachable with same assumptions

# Datar – Mathews model (2004)




Result presented as a single number

# Pay-off method for real option valuation (2009)

- Manager created cash-flow scenarios are used to create (simplified) future value distributions that are treated as fuzzy numbers (often triangular / trapezoidal)
- The ”expected value” is calculated from the fuzzy numbers as the possibilistic mean of the positive side of the fuzzy number
- Discounting is done by using separate discount rates for revenues and costs
- compounding interval at the discretion of the user
- No strict assumptions about the markets / reality
- Spread-sheet compatible
- Designed for the practitioner – ”as easy as possible”

# Pay-Off Method Shortly

1. Build 3-4 cash-flow scenarios and perform present value & NPV calculations
2. Create the pay-off distribution
3. Calculate descriptives for additional decision- support

The method of calculation can also be ”less” standard. Many times cash-flows cannot be estimated, only ”larger” aggregates are available.

| **Strategic patent portfolio 1** | Min possible | Best estimate | Max possible |
|---|---|---|---|
| Total size of future markets ($) | $900M | $1700M | $2900M |
| Market share (%) | 6% | 15% | 24% |
| Likelihood of enabling portfolio (%) | 30% | 40% | 55% |
| Overall chances (%) | 75% | 90% | 100% |
| $V_{Portfolio}$ = TSFM*EMS*LBEP*RT | $12,15M | $306M | $382,8M |
| PV total costs of portfolio | $8M | $14M | $35M |
| $NPV_{portfolio}$ | $-22,85M | $292M | $374,8M |

We ask for three scenarios:

1. Best estimate
2. minimum possible
3. maximum possible

These can be fully unrelated or fully related to each other

**CUMULATIVE NET PRESENT CASH-FLOW SCENARIOS FOR THE ASSET**

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| maximum | -800,00 | -452,17 | -149,72 | 113,29 | 341,99 | 540,86 | 713,79 |
| Best est. | -990,00 | -798,70 | -632,34 | -467,96 | -325,03 | -200,73 | -92,65 |
| minimum | -1000,00 | -913,04 | -761,81 | -630,31 | -515,96 | -416,53 | -330,06 |

**”EACH SCENARIO IS JUST TRADITIONAL NPV ”**

**”Everyone in business knows this!”**

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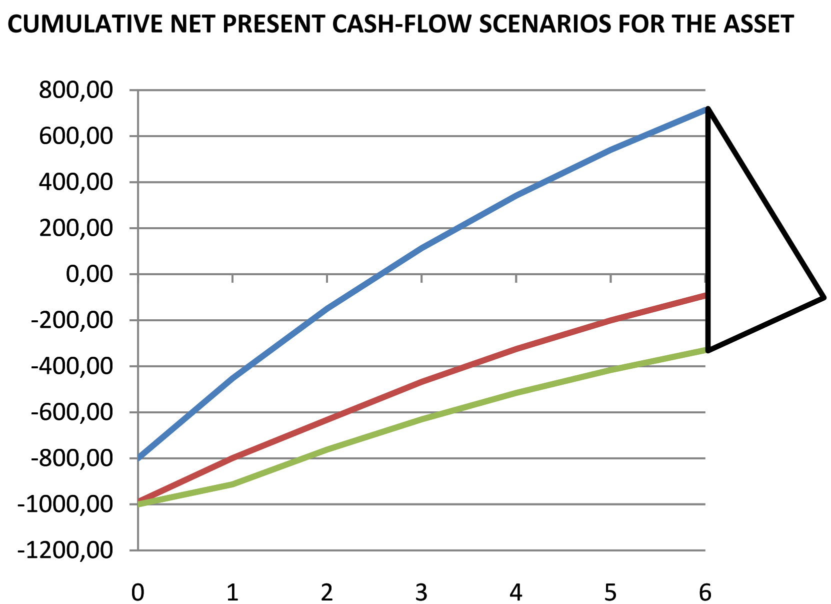


| CUMULATIVE NET PRESENT CASH-FLOW SCENARIOS FOR THE ASSET | | | | | | | |
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# Pay-Off Method Shortly

1 Build 3-4 cash-flow scenarios and perform present value & NPV calculations

2 Create the pay-off distribution

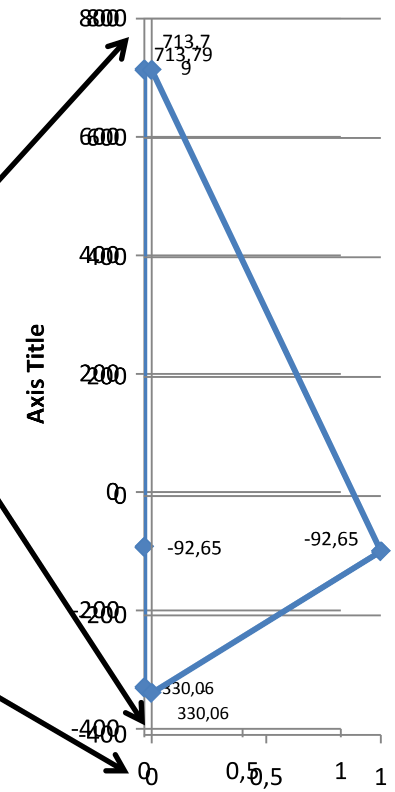
3 Calculate descriptives for additional decision- support

It is not suggested that this is a perfect reflection of reality, or that this would be a very accurate description of reality. This is a way to **graphically** represent inaccurate knowledge that **certainly looks very interesting**!

The three scenarios' for the NPV of the asset mapped on a $ ( vertical ) axis.

We do not consider outcomes outside the min and max possible!

On the horizontal axis we map the degree of membership in the (set of) possible outcomes.
1 = fully possible outcome
0 = not a possible outcome




The most likely scenario is assigned full membership in the set of possible outcomes.

As a result we have created a distribution of the NPV of the asset, *based on the Information that we have.*

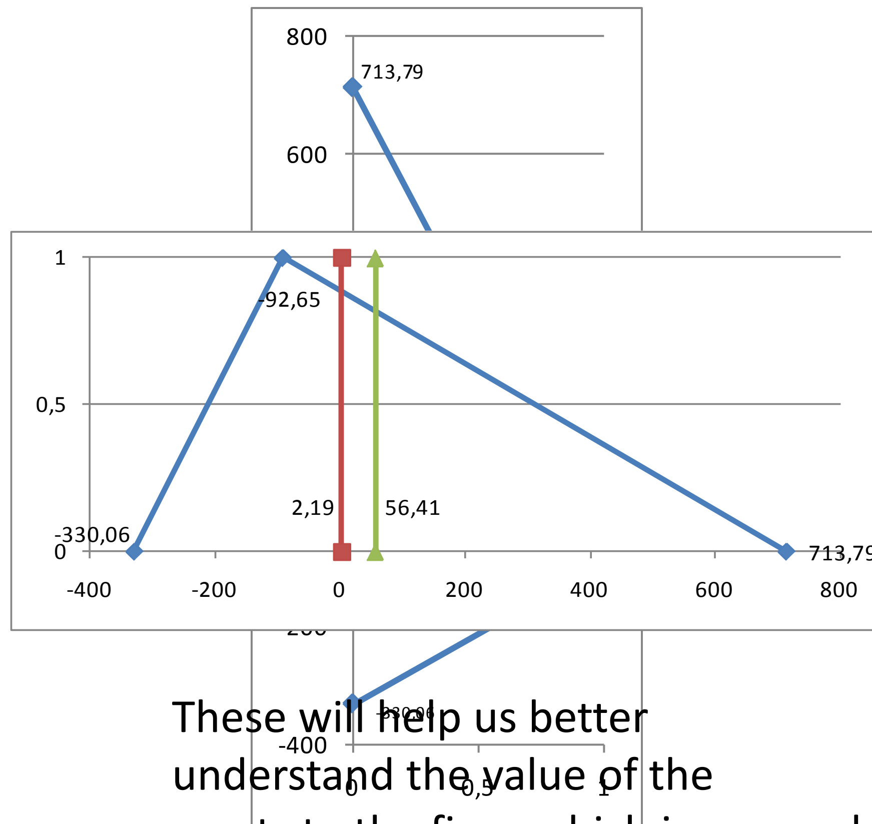
# Pay-Off Method Shortly

1. Build 3-4 cash-flow scenarios and perform present value & NPV calculations
2. Create the pay-off distribution
3. Calculate descriptives for additional decision- support

*<u>We can treat the resulting distribution as a fuzzy number</u>*. This allows us to calculate interesting descriptive numbers directly from it.

With simple calculations we can calculate, for example: a **real option value** and a (possibilistic) mean for the asset.

”If we turn the problem” and look at it from a different angle...




These will help us better understand the value of the assets to the firm, which is our goal.

# Real Option Valuation based on a pay-off distribution

$$ROV = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} \times E(A_+)$$

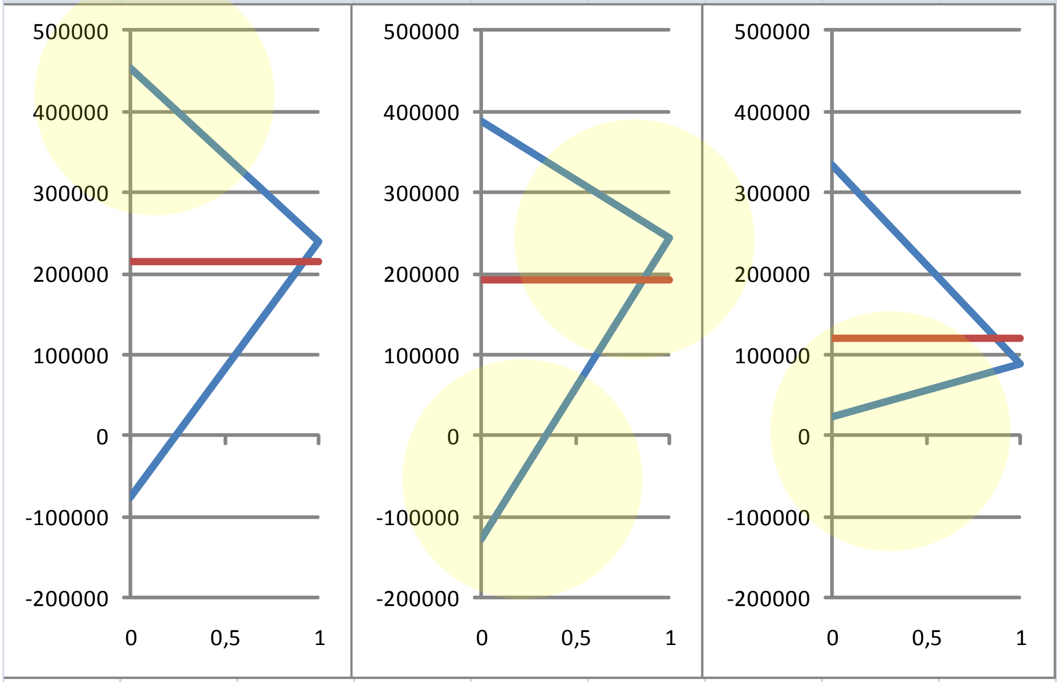
Possibilistic mean of the positive side of the distribution.

Simple formulae, used based on where zero ”crosses” the distribution

$$E(A_+) = \begin{cases} a - \alpha > 0 \; then \; E(A_+) = \frac{2a}{2} + \frac{\beta - \alpha}{6} \\ a > 0 > a - \alpha \; then \; E(A_+) = \frac{(\alpha - a)^3}{6\alpha^2} + a + \frac{\beta - \alpha}{6} \\ 0 > a \; then \; E(A_+) = \frac{(a - \beta)^3}{6\beta^2} \\ a + \beta < 0 \; then \; E(A_+) = 0 \end{cases}$$

# Comparison of assets becomes more intuitively understandable

| ASSET 1 | | ASSET 2 | | ASSET 3 | |
|---|---|---|---|---|---|




| | | | | | |
|---|---|---|---|---|---|
| Best guess | 238850 | Best guess | 245000 | Best guess | 89650 |
| Optimistic | 454400 | Optimistic | 387400 | Optimistic | 333650 |
| Pessimistic | -76050 | Pessimistic | -127600 | Pessimistic | 23544 |
| | | | | | |
| ROV | 215309 | ROV | 191383 | ROV | 119299 |
| Single # NPV | 222292 | Single # NPV | 206633 | Single # NPV | 119299 |

The decision-maker will have to make a "real" decision = selection!

**Focus on potential!**
**Focus on downside!**
**Quantifying uncertainty !**

# Models summary

| | Process to create future value distribution | Calculation of expected value | Discounting | Other |
|---|---|---|---|---|
| Black & Scholes | GBM, continuous log normal | Probability weighted average | Continuous with rf (same for R & C) | Closed form solution, replication |
| Binomial tree model | Binomial tree, quasi log normal | Probability weighted average | Compound interval the time step used, risk free rate | Backwards iteration to solve value, approaches B&S |
| Datar – Mathews | Managerial CF scenarios => Monte Carlo | Probability weighted average | Separate rd for cost & revenue, c. interval selectable | Practitioner oriented |
| Fuzzy pay-off method | Managerial CF scenarios => fuzzy number | Possibilistic mean value | Separate rd for cost & revenue, c. Interval selectable | Simple to use, fuzzy logic |

# Thoughts about future research

# Thoughts about future research

Using **subjective probability** in framing the problem

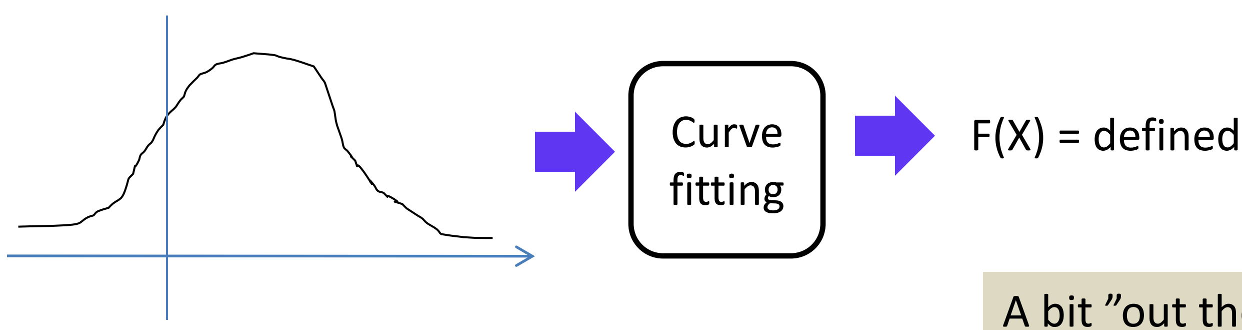
- similar ease of use obtainable as with the fuzzy pay-off method, but with probability theory

- same methods for creation of cash-flow scenarios etc. apply – just a different choice of theory for treatment of uncertainty

# Thoughts about future research

Using **function recognition** / estimation for deriving a ”high fit” function (curve) for a distribution based on managerial information

- managers draw a future value distribution and curve is fitted to allow ”normal” probability distribution based real option valuation




A bit ”out there” approach

# Thoughts about future research

Presentation of results is a KEY factor in successfull use of real options!

And mind you ANY OR Results!!!

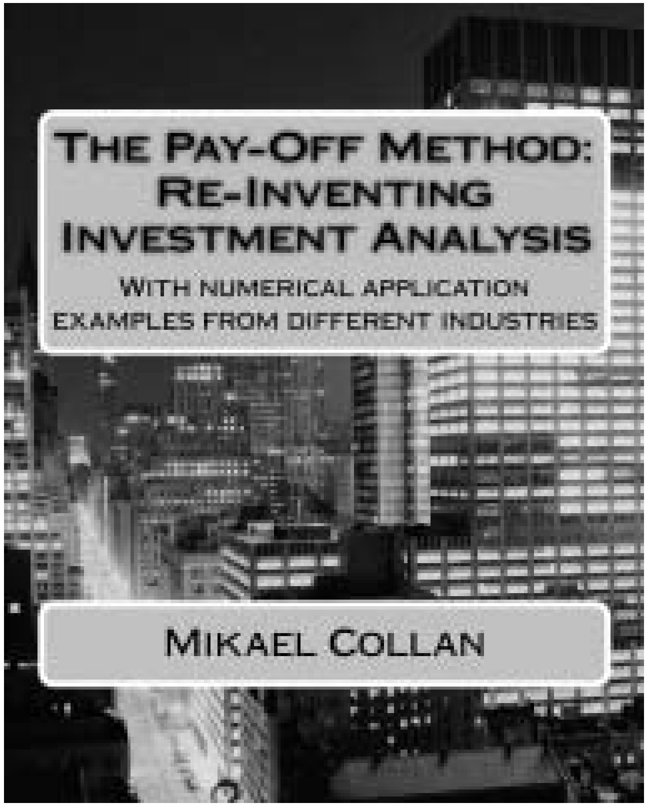
Research into *how results are presented* **”does not exist”** in connection with real option valuation

This is something that I see as a modellers’ failure – only the valuation is modelled, but presenting the results is left for others to ponder.

# Final thoughts

- I am quite sure that there are numerous new possibilities to model real option valuation – the problem is good and relevant and there is a real need from the practitioners
- Presentation of results is also something that merits the modelers' time => This is AS IMPORTANT as the model itself, perhaps even more important!
- **How much time did you ever think about presentation of your results?**

# New book out on the pay-off method!



# Thank you!

**Comments?**
**Observations?**
**Questions?**









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# References


[1] Bachelier L 1900 Théorie de la spéculation. *Annales Scientifiques de l'École Normale Superieure* **17** 21-86
[2] Black F and Scholes M 1973 The Pricing of Options and Corporate Liabilities *Journal of Political Economy* **81** 637-59
[3] Borison A 2005 Real Options Analysis: Where Are the Emperor's Clothes? *Journal of Applied Corporate Finance* **17** 17-31
[4] Boyle P 1977 Options: A Monte Carlo approach *Journal of Financial Economics* **4** 323-8
[5] Boyle P 1986 Option Valuation Using a Three-Jump Process *International Options Journal* **3** 7-12
[6] Buckley J and Esfandiar E 2008 Pricing Stock Options Using Black-Scholes and Fuzzy Sets *New Mathematics and Natural Computation* **4** 165–76
[7] Carlsson C, Fuller R, Heikkilä M and Majlender P 2007 A fuzzy approach to R&D project portfolio selection *International Journal of Approximate Reasoning* **44** 93-105
[8] Collan M 2004 Giga-Investments: Modelling the Valuation of Very Large Industrial Real Investments. In: *Department of Information Systems,* (Turku: Abo Akademi University)
[9] Collan M, Carlsson C and Majlender P 2003 Fuzzy Black and Scholes Real Option Pricing *Journal of Decision Systems* **12** 391-416
[10] Collan M, Fullér R and Mézei J 2009 Fuzzy Pay-off Method for Real Option Valuation *Journal of Applied Mathematics and Decision Systems* **2009**
[11] Courtault J-M 2000 Louis bachelier - On the centenary of théorie de la spéculation *Mathematical Finance* **10** 341-53
[12] Cox J, Ross S and Rubinstein M 1979 Option Pricing: A Simplified Approach *Journal of Financial Economics* **7** 229-63
[13] Datar V and Mathews S 2004 European Real Options: An Intuitive Algorithm for the Black Scholes Formula *Journal of Applied Finance* **14**
[14] Datar V, Mathews S and Johnson B 2007 A Practical Method for Valuing Real Options: The Boeing Approach *Journal of Applied Corporate Finance* **19** 95-104
[15] Haug E and Taleb N 2009 Why have we never used the Black-Scholes-Merton Option Pricing Formula. In: *Available at SSRN: [http://ssrn.com/abstract=101207](http://ssrn.com/abstract=101207),*
[16] Keppo J and Lu H 2003 Real options and large producer: the case of electricity markets *Energy Economics* **25** 459-72
[17] Kulatilaka N and Marcus A 1992 Project valuation under uncertainty: When does DCF fail? *Journal of Applied Corporate Finance* **5** 92-100
[18] Leslie K and Michaels M 1997 The real power of real options *The McKinsey Quarterly* 5-22
[19] Lint O and Pennings E 1998 R&D as an option on market introduction *R&D and Management* **28** 279-87
[20] Mathews S and Datar V 2007 A Practical Method for Valuing Real Options: The Boeing Approach *Journal of Applied Corporate Finance* **19** 95-104
[21] Mathews S, Datar V, Nakamoto K and Forgie C 2006 System, method and computer program product for determining a minimum asset value for exercising a contingent claim of an option ed T B Company" (USA
[22] Merton R 1973 Theory of Rational Option Pricing *Bell Journal of Economics and Management Science* **4** 141-83
[23] Muzzioli S and Reynaerts H 2008 American optionnext term pricing with imprecise risk-neutral probabilities *International Journal of Approximate Reasoning* **49** 140-7
[24] Muzzioli S and Torricelli C 2000 Combining the Theory of Evidence with Fuzzy Sets for Binomial Option Pricing. In: *Materiale di Discussione* Dipartimento di Economia Politica, Universita degli Studi di Modena e Reggio Emilia)
[25] Muzzioli S and Torricelli C 2000 A model for pricing an option with a fuzzy payoff *Fuzzy Economics Review* **6**
[26] Myers S 1977 Determinants of corporate borrowing *Journal of Financial Economics* **5** 146-75
[27] Newton D and Pearson A 1994 Application of option pricing theory to R&D *R&D Management* **24** 83-9
[28] Sick G 1990 *Capital Budgeting with Real Options* (New York: Stern School of Business, New York University)
[29] Tolga C and Kahraman C 2008 Fuzzy Multiattribute Evaluation of R&D Projects Using a Real Options Valuation Model *International Journal of Intellige Systems* **23** 1153-76
[30] Zmeskal Z 2001 Application of the fuzzy-stochastic methodology to appraising the firm value as a European call option *European Journal of Operational Research* **135** 303-10