COMPOUND REAL OPTIONS WITH THE FUZZY PAY-OFF METHOD

Three-stage Case Illustration

Mikael Collan

IAMSR
Institute for Advanced Management Systems Research
at Åbo Akademi University

This Presentation

- Background
- Fuzzy pay-off method for real option valuation
- Compound real options
- Three-stage case illustration
- Conclusions

Background

- Compound options are found in many different types of investments which makes their valuation an interesting and important research topic
- Compound real options are more complex than ”one period” real options; the existing models for valuation of compound real options are mathematically demanding
- Complexity of real option valuation makes valuation of compound options often a black box for practitioners => this is a problem because real options are often a very usable tool for compound options, e.g., in R&D – but practitioners often decide not to utilize real option valuation because it is not easy to understand
- Fuzzy pay-off method for real option valuation is a new simple-to-understand method for real option valuation that can also be used to frame compound option valuation problems

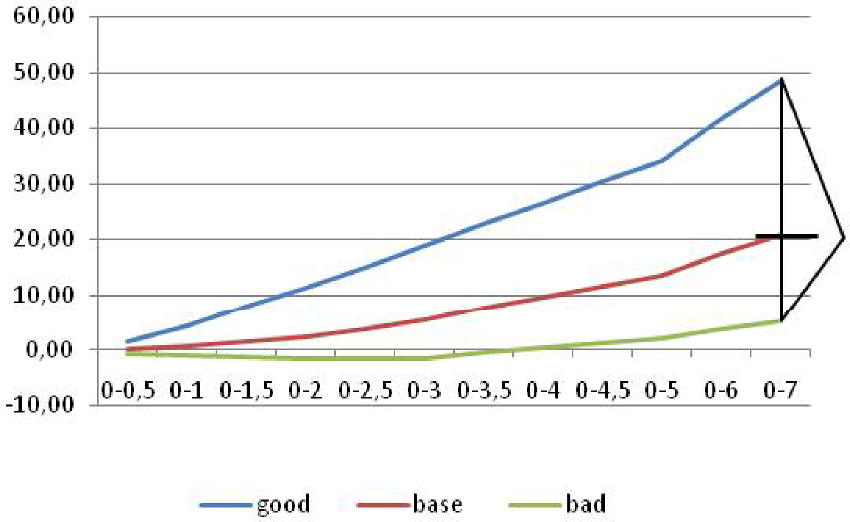
Fuzzy pay-off method for real option valuation (FPOM)

- Fuzzy pay-off method is a new method for calculating the value of a real option
- Usable when we want to use scenario type cash-flows as the basis for real option valuation
- Based on fuzzy numbers & fuzzy logic
- Simple to use and to understand
- Published in:

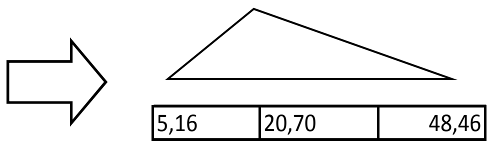
Collan, M., Fullér, R., & Mézei, J. 2009. Fuzzy Pay-off Method for Real Option Valuation. Journal of Applied Mathematics and Decision Systems, 2009.

Fuzzy pay-off method for real option valuation (FPOM)

- A fuzzy NPV is created from project cash-flow scenarios
- This is also the pay-off distribution from the project



Fuzzy NPV is an expected NPV distribution for the project

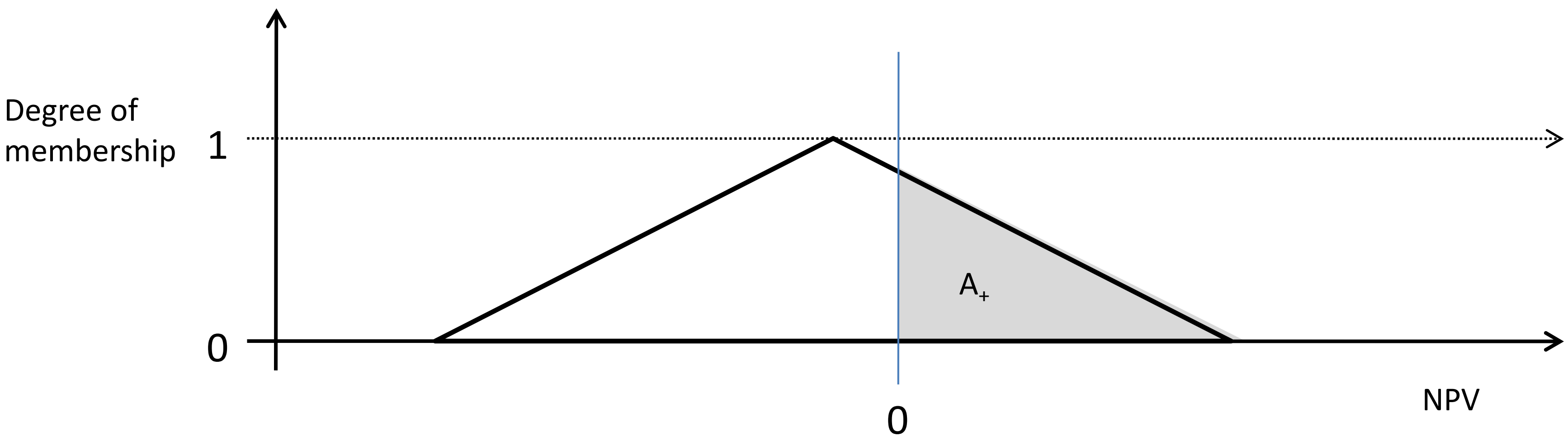


The distribution can be of any shape, but for usability triangular or trapezoidal fuzzy numbers are often used.

An important point in using this method is to agree on how the resulting pay-off distribution from the scenario cash-flow data is generated.

Fuzzy pay-off method for real option valuation (FPOM)

- Real option value is the fuzzy mean of the positive side of the fuzzy NPV multiplied by the area above the positive values divided by the total area of the fuzzy NPV.



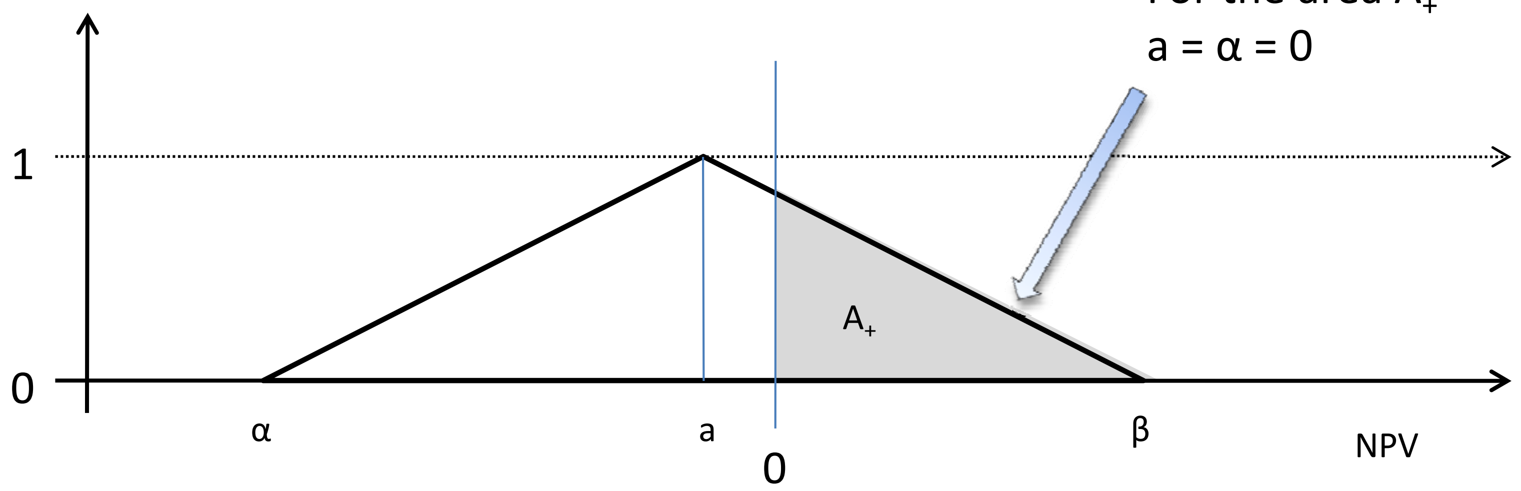
$$ROV = \frac{\int_0^{\infty} A(x)dx}{\int_{-\infty}^{\infty} A(x)dx} \times E(A_+)$$

The trick is to know how to calculate the mean

Fuzzy pay-off method for real option valuation (FPOM)

- Calculation of possibilistic mean is defined in
  Carlsson, C. & Fullér, R. 2001. On possibilistic mean value and variance of fuzzy numbers. Fuzzy Sets and Systems, 122: 315-326.
- Possibilistic mean for triangular fuzzy numbers

$$E(A_+) = \frac{(\alpha - a)^3}{6\alpha^2} + a + \frac{\beta - \alpha}{6}.$$



Fuzzy pay-off method for real option valuation (FPOM)

$$I_1+I_2=\int_0^{z_1}\gamma(a-\alpha+z+a+(1-\gamma)\beta)\mathrm{d}\gamma+\int_{z_1}^{1}\gamma(a-(1-\gamma)\alpha+a+(1-\gamma)\beta)\mathrm{d}\gamma$$

where

$$z_1=1-\frac{\alpha-z}{\alpha}=\frac{z}{\alpha}$$

$$I_1=\int_0^{z_1}[(2a-\alpha+z+\beta)\gamma-\gamma^2\beta]\mathrm{d}\gamma=$$

$$(2a-\alpha+z+\beta)\times\frac{z^2}{2\alpha^2}-\beta\times\frac{z^3}{3\alpha^3}$$

$$I_2=\int_{z_1}^{1}(2a+\beta-\alpha)\gamma\mathrm{d}\gamma-\int_{z_1}^{1}(\beta-\alpha)\gamma^2\mathrm{d}\gamma=$$

$$(2a-\alpha+\beta)\int_{z_1}^{1}\gamma\mathrm{d}\gamma-(\beta-\alpha)\int_{z_1}^{1}\gamma^2\mathrm{d}\gamma=$$

$$(2a-\alpha+\beta)\left(\frac{1}{2}-\frac{z^2}{2\alpha^2}\right)-(\beta-\alpha)\left(\frac{1}{3}-\frac{z^3}{3\alpha^3}\right)$$

$$I_1+I_2=(2a-\alpha+z+\beta)\times\frac{z^2}{2\alpha^2}-\beta\times\frac{z^3}{3\alpha^3}+(2a-\alpha+\beta)\left(\frac{1}{2}-\frac{z^2}{2\alpha^2}\right)-(\beta-\alpha)\left(\frac{1}{3}-\frac{z^3}{3\alpha^3}\right)$$

$$I_1+I_2=\frac{z^3}{2\alpha^2}+\frac{2a-\alpha+\beta}{2}+\frac{\alpha-\beta}{3}-\alpha\frac{z^3}{3\alpha^3}$$

$$\boxed{E=\frac{z^3}{6\alpha^2}+a+\frac{\beta-\alpha}{6}}$$

Compound real options

- Compound real options are commonly found in real investments; investments that open other investments or actions within projects that open the way for new actions are usual
- Strategic investments are usually investments that are ”door openers” rather than per se highly profitable
- R&D investments and application of their results can almost always be understood as compound real options
- Valuation of strategies as real options requires compound real option valuation
- The models used for valuation of compound options are most often based on Geske 1979

  - based on Black-Scholes

  - quite difficult to use (black box for practitioners)

  - assumptions very restrictive

We show with a simple case how the fuzzy pay-off method can be applied to more easily and understadably value compound real options

Three-stage case

- We are looking at a three-stage R&D investment problem with two stages of R&D and then a possible production investment

- The license to start the R&D is for sale for 50 monetary units? Buy or not?

Three-stage case

<u>First stage</u> - initial investment into exploratory R&D

The exploratory R&D duration is uncertain and we simplify it to three (min, best guess, max) scenarios 3, 5, and 7 units of time, we assume that there is a fixed cost (500 mu) per unit of time.

<u>Second stage</u> - investment into research in application

The application R&D duration is uncertain and we simplify it to three scenarios 1, 2, and 3 units of time, we assume that there is a fixed cost (1000 mu) per unit of time,

<u>Third stage</u> - production investment and operation of the investment

The production investment has a fixed cost (20000 mu) and will be made in the period directly after the completion of the application R&D and will have the duration of 1 unit of time. The sales of the product are estimated to last for 5 years. Three scenarios for operational cash-flow are estimated, for simplicity the scenarios are ±20% from the best guess scenario

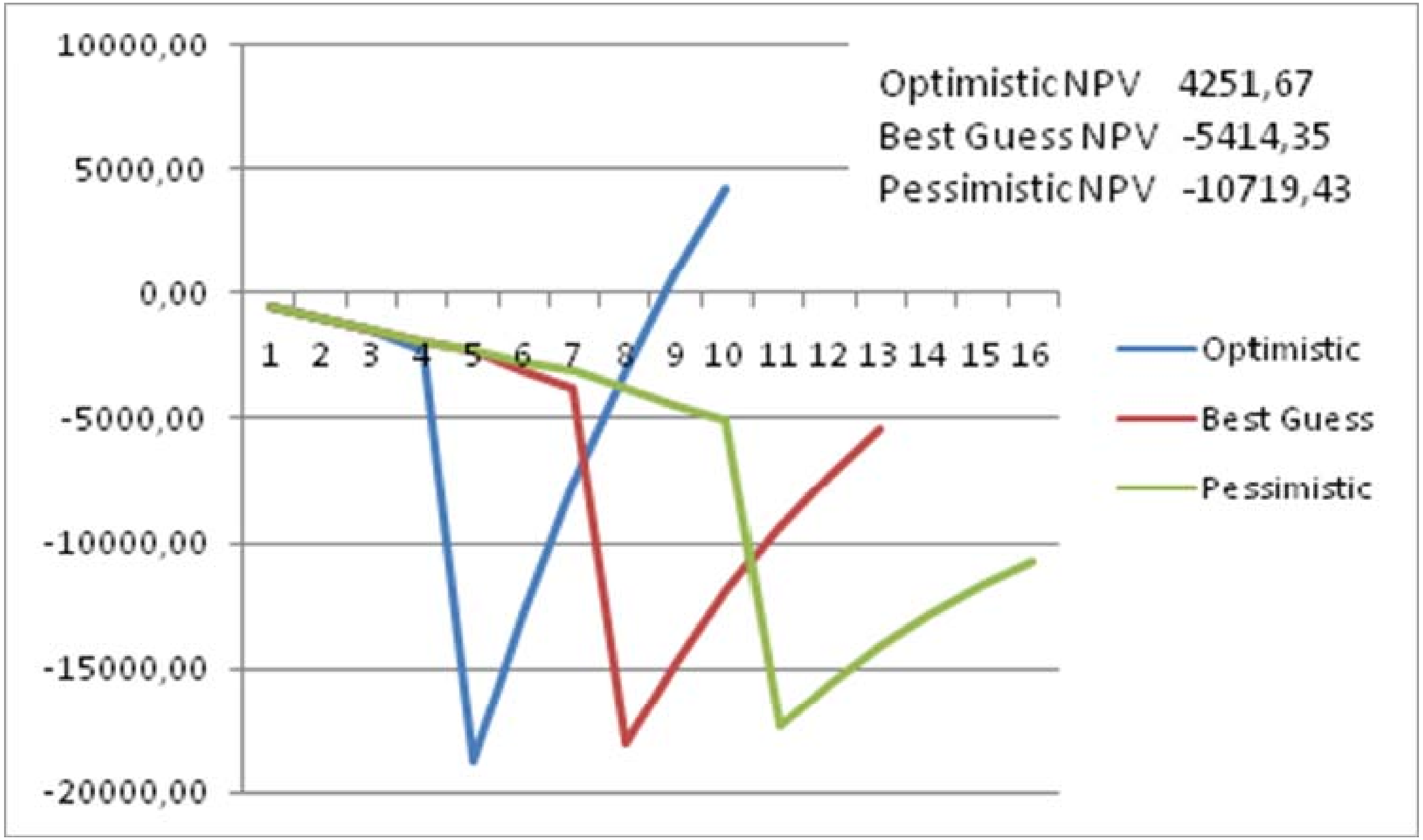
Three-stage case

CF	-500	-500	-500	-1000	-20000	12000	12000	12000	12000	12000						
CF	-500	-500	-500	-500	-500	-1000	-1000	-20000	10000	10000	10000	10000	10000			
CF	-500	-500	-500	-500	-500	-500	-500	-1000	-1000	-1000	-20000	8000	8000	8000	8000	8000
PV	-500,00	-476,19	-453,51	-863,84	-16454,05	5966,12	5187,93	4511,24	3922,82	3411,15						
PV	-500,00	-476,19	-453,51	-431,92	-411,35	-783,53	-746,22	-14213,63	3269,02	2842,62	2471,85	2149,43	1869,07	0,00	0,00	0,00
PV	-500,00	-476,19	-453,51	-431,92	-411,35	-391,76	-373,11	-710,68	-676,84	-644,61	-12278,27	1719,55	1495,26	1300,22	1130,63	983,16
NPV	4251,67															
NPV	-5414,35															
NPV	-10719,43															
time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0,05	discount rate for investments														
	0,15	discount rate for operational income CF														
Cum. PV	-500,00	-976,19	-1429,71	-2293,54	-18747,59	-12781,47	-7593,54	-3082,30	840,53	4251,67						
Cum. PV	-500,00	-976,19	-1429,71	-1861,62	-2272,98	-3056,50	-3802,72	-18016,34	-14747,33	-11904,70	-9432,85	-7283,42	-5414,35			
Cum. PV	-500,00	-976,19	-1429,71	-1861,62	-2272,98	-2664,74	-3037,85	-3748,53	-4425,37	-5069,98	-17348,24	-15628,69	-14133,44	-12833,21	-11702,58	-10719,43

Cash-flow table for the three-stage project

Cumulative NPV for the three scenarios

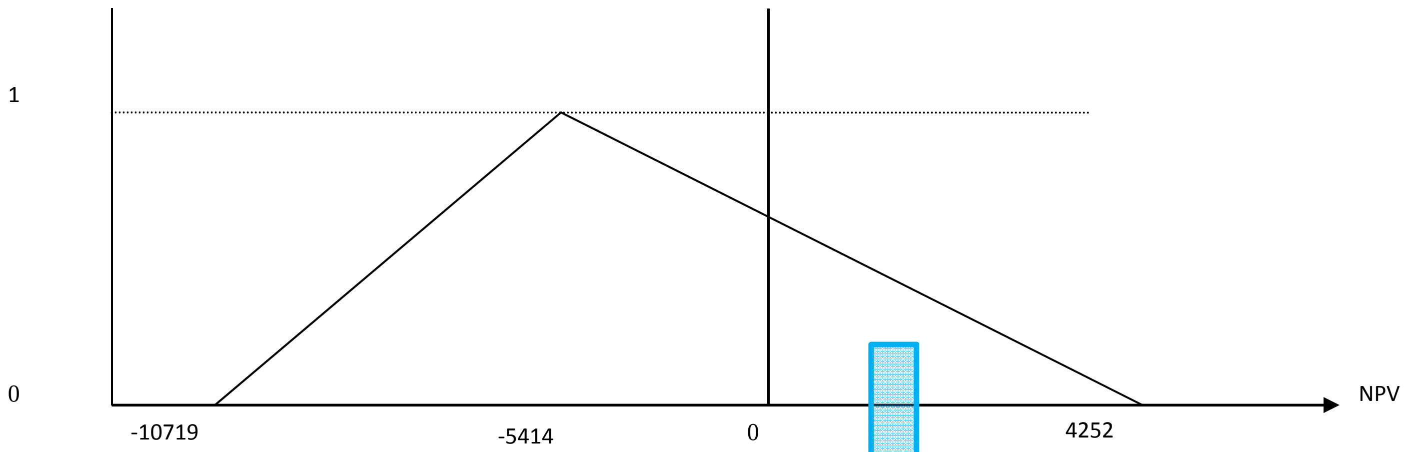
Three-stage case



Cumulative NPV for the three scenarios

Three-stage case

Triangular fuzzy distribution for the expected project NPV (figure not in scale)



$$E(A_+) = \frac{(\alpha - a)^3}{6\alpha^2} + a + \frac{\beta - \alpha}{6}.$$

ROV = 89 mu

Cost 50 mu

Cost < ROV => decision: invest (buy)

Three-stage case: Milestone evaluation

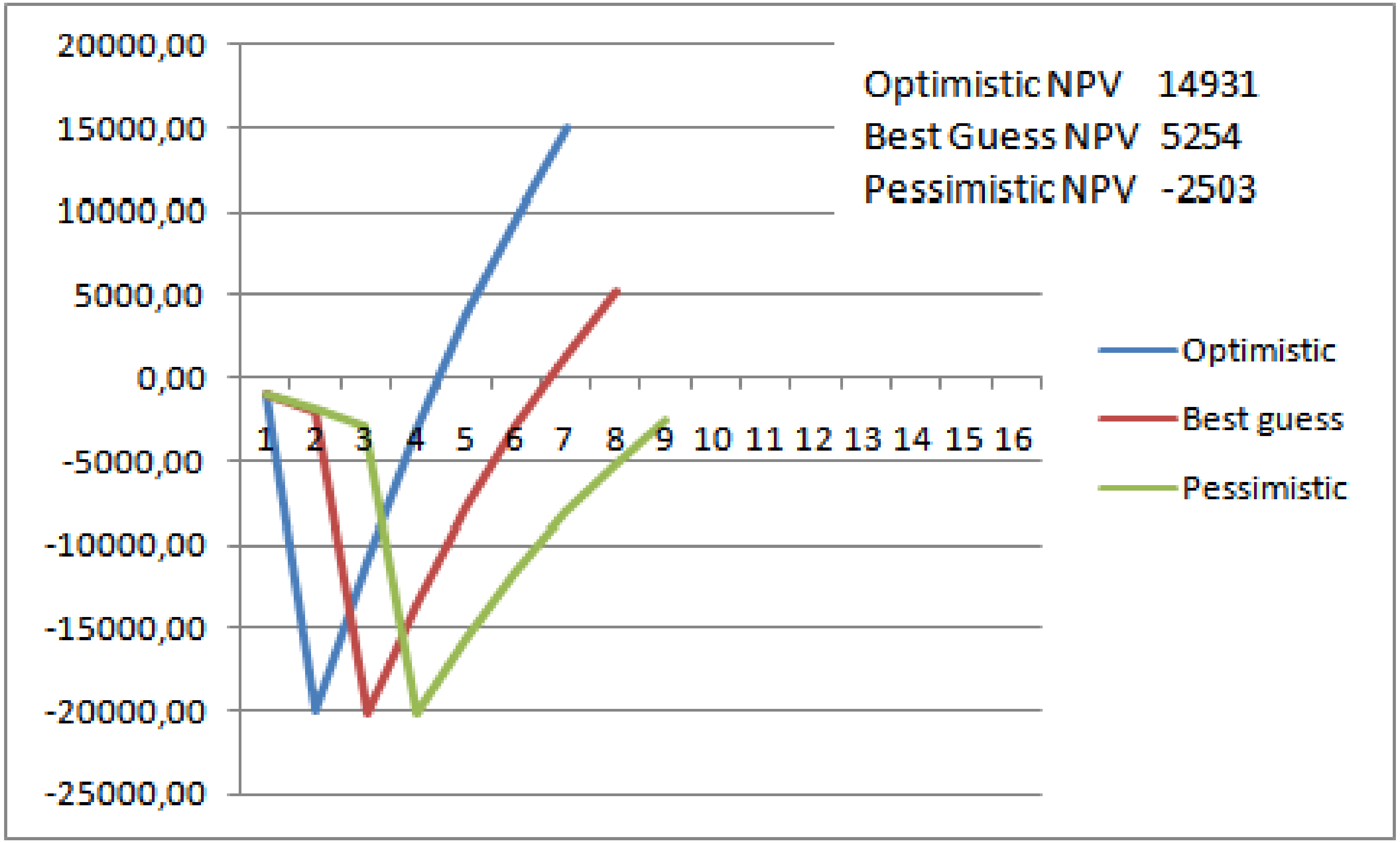
- It is very common that after an investment decision is made and a project is started the project is re-evaluated from time to time (especially common for R&D investments)
- This is done to see if the future expectations have changed – and how much and to which direction they have changed: new information is added into the analysis
- Changed expectations about the future translated into changes in the cash-flow tables and the pay-off distribution => changed ROV

Three-stage case: Milestone evaluation

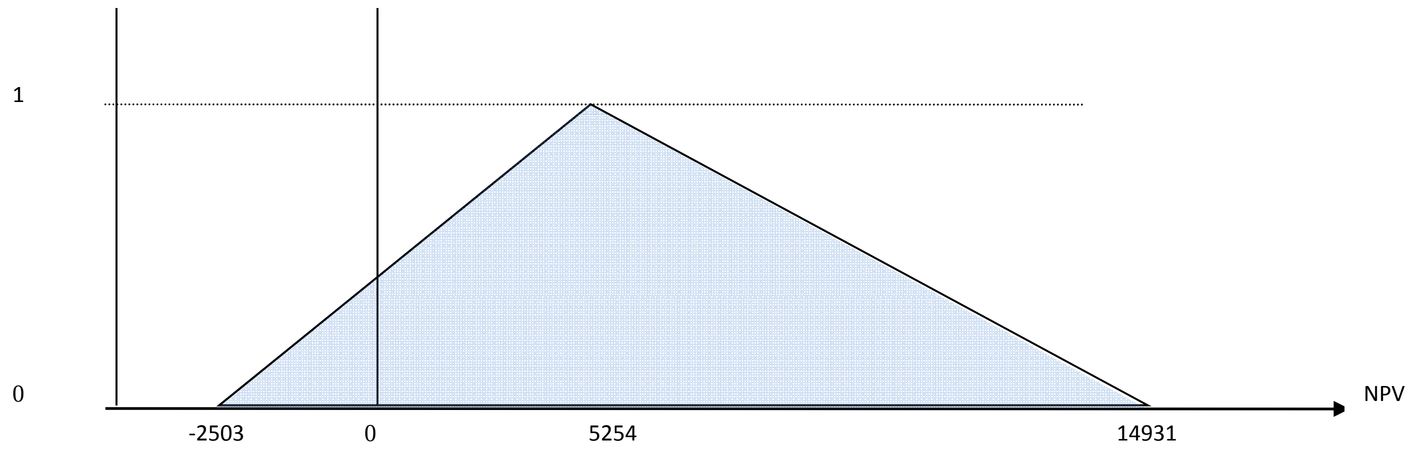
CF	-1000	-20000	12000	12000	12000	12000	12000									
CF	-1000	-1000	-20000	10000	10000	10000	10000	10000								
CF	-1000	-1000	-1000	-20000	8000	8000	8000	8000	8000							
PV	-1000,00	-19047,62	9073,72	7890,19	6861,04	5966,12	5187,93	0,00	0,00	0,00						
PV	-1000,00	-952,38	-18140,59	6575,16	5717,53	4971,77	4323,28	3759,37	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PV	-1000,00	-952,38	-907,03	-17276,75	4574,03	3977,41	3458,62	3007,50	2615,21	0,00	0,00	0,00	0,00	0,00	0,00	0,00
NPV	14931,39															
NPV	5254,14															
NPV	-2503,39															
time	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0,05	Discount rate for costs														
	0,15	Discount rate for operational income CF														
Cum. PV	-1000,00	-20047,62	-10973,90	-3083,70	3777,34	9743,46	14931,39									
Cum. PV	-1000,00	-1952,38	-20092,97	-13517,81	-7800,28	-2828,51	1494,77	5254,14								
Cum. PV	-1000,00	-1952,38	-2859,41	-20136,16	-15562,14	-11584,72	-8126,10	-5118,61	-2503,39							

Cash-flow table for the project at first milestone evaluation, after the first stage R&D is completed (two-stage project remaining)
Here with these numbers we assume that the cash-flows are unchanged.

Three-stage case: Milestone evaluation



Three-stage case: Milestone evaluation



The milestone evaluation in this case includes the revenues and the costs, so the ROV is comparable to a cost of zero => positive ROV means that the project should be continued.

New information will change the pay-off distribution to the better...

or to the worse.

The decision maker will adopt the decision according to the information available.

Conclusions

The FPOM method can easily accomodate:

i) Different underlying processes in different project stages

ii) Different timing scenarios for different project stages

iii) The integration of independent project stage scenarios into project scenarios

iiii) Milestone evaluation with changed information

References

- Carlsson, C. & Fullér, R. 2001. On possibilistic mean value and variance of fuzzy numbers. Fuzzy Sets and Systems, 122: 315-326.
- Collan, M., Fullér, R., & Mézei, J. 2009. Fuzzy Pay-off Method for Real Option Valuation. Journal of Applied Mathematics and Decision Systems, 2009.

  available at: http://www.hindawi.com/journals/jamds/aip.238196.html
- Geske, R. 1979. The Valuation of Compound Options. Journal of Financial Economics, 7(1): 63–81